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ABSTRACT

The re-entry into a planetary atmosphere of a body of any form is considered. The body is assumed to be thermally anisotropic and the thermal coefficients are assumed to vary with temperature.

A general procedure is introduced to study the thermal transient due to kinetic heating and radiation. Some approximate similarity laws for heat conduction in structures of small thickness and numerical examples complete the work.

1. INTRODUCTION

It is possible to substantially lessen the heat shidding problem for spaceships re-entering into the terrestrial atmosphere if the material employed for the shidding structures exhibits a suitable heat conduction anisotropy. For this purpose the material shall have a thermal conductivity in an established direction much less than in any other direction normal to the first (Ref. 1).

Many new materials as, for an example, the pyrolytic graphite exhibits such anisotropy. It is, also, important to point out that the corresponding anisotropy tensor can strongly vary in function of the temperature (Ref. 2).

The corresponding problem of the heat conduction can be formulated, indeed, by the following equations (Ref. 3):

$$\text{at } t = t_0 \quad T = T_0 \quad (1)$$

$$\text{at } t > t_0 \quad - \underline{f} = \underline{K} \cdot \text{grad } T \quad (2)$$

$$- \text{div } \underline{f} + q = c \frac{\delta T}{\delta t} \quad (3)$$

$$- \underline{f} \cdot \underline{\nu} = Q \quad (4)$$

In the abovesaid equations T_0 is the initial temperature at time $t=t_0$.

The components of the two vectors $\text{grad } T$ and \underline{f} along a whichever direction give respectively the derivative of the temperature T and the heat flux in that direction. They are related through a linear transform (2) operated by the conductivity tensor \underline{K} .

In any element of unity volume the heat conduction flux clearing the unity time is $-\text{div } \underline{f}$, while the heat generated inside the elements is q , calling c specific heat. The component of $-\underline{f}$ along the unit vector $\underline{\nu}$ normal to external surface of the body σ , represents the heat flux Q for unit of time and for unit of surface received by the body through σ . It is assumed that in equations (3) and (4) one can write q and Q in function of the

other quantities which are defining the problem (Ref. 4).

Normally the system of equations (1), (2), (3) and (4) is not linear with T . The \underline{K} and c , indeed, are generally functions of T . Moreover the functional dependence of Q with T may be not linear. The independent variables of the problem are in general four: the three spacewise coordinates plus the time wise coordinate. These premises show very clearly from one side the difficulties to obtain exact analytic solutions and from the other the complexity of an exclusively numerical approach.

The present report deals basically with the following points:

- i) The definition of an ideal material called hereinafter U.A.L.D. (Uniformly Anisotropic with Linear Diffusivity) and it can be considered as a model of new materials of practical interest. This definition is very well consistent with available experimental data.
- ii) An integration procedure of the (1), (2), (3) and (4) equations applied to a body made of whichever U.A.L.D. material. By using the proposed integration procedure, the abovesaid equations are substituted by a small number of first order integral Volterra equations depending only upon time.
- iii) Some approximate similarity laws for heat conduction in structures of small thickness.
- iiii) Numerical examples.

2. THE UNIFORMLY ANISOTROPIC BODY

It is considering an anisotropic body. In a point of the body, \underline{K} is the heat conductivity tensor and c the specific heat. Letting T_0 a reference temperature, \underline{K}_0 and c_0 are the corresponding values of \underline{K} and c for $T=T_0$. Now it will be defined as uniformly anisotropic a body where at each point and for any value of T , the tensor \underline{K} can be obtained as the product of a scalar function $A(T)$ by the tensor \underline{K}_0 . In view of the fact that \underline{K} is a symmetrical second order tensor, there are three orthogonal principal directions and three principal values of the conductivity. It follows that in a point of the body where:

$$(5) \quad \underline{K}(T) = A(T) \cdot \underline{K}_0 \quad (\text{body uniformly anisotropic})$$

- i) the three principal direction do not vary with temperature;
- ii) the three principal conductivities are varying proportionally.

Subsequently it will be assumed that the two temperature functions $A(T)$ and $\frac{c(T)}{c_0}$ be the same for all the points of the body. If the body is homogeneous c_0 and K_0 will be also the same in any point. Finally the body homogeneous isotropic when point by point c_0 is the same and K_0 rotates its own three principal directions along the coordinate lines of a orthogonal curvilinear reference system (e.g. cylindrical, polar spherical etc.), keeping unvaried the three principal values of the conductivity.

The abovesaid case is particularly important for the new materials which are of interest for the present report.

Remembering now that in equation (5) $A(T_0) = 1$ and introducing equation (5) in equation (1) and (2), it obtains:

$$\operatorname{div} \left[\underline{K}_0 A(T) \cdot \operatorname{grad} T \right] + q = \frac{c}{c_0} c_0 \frac{\delta T}{\delta t} \quad (6)$$

Now two new temperatures T' and T'' are defined as it follows:

$$T' = \int_{T_0}^T A(T) \cdot dT \quad (7)$$

$$T'' = \int_{T_0}^T \frac{c(T)}{c_0} \cdot dT \quad (8)$$

If the thermal characteristics of the body were not varying with temperature, the following results could be obtained:

$$A = \frac{c}{c_0} = 1 \quad \text{therefore} \quad T'' = T' = T - T_0$$

In the general case it can be written:

$$T'' = T' + b_2 (T')^2 + b_3 (T')^3 + \dots \quad (9)$$

where b_2, b_3 are constant.

Introducing now (7) and (8) into (6) it comes out:

$$\operatorname{div} \left[\underline{K}_0 \cdot \operatorname{grad} T' \right] + q = c_0 \frac{\delta T''}{\delta t} \quad (10)$$

Substituting the (9) into (10) it obtains: (11)

$$\operatorname{div} \left[\underline{K}_0 \cdot \operatorname{grad} T' \right] + q = c_0 \frac{\delta T'}{\delta t} \left[1 + 2b_2 T' + 3b_3 T'^2 \dots \right]$$

The eq. (11) can be considered as the conductivity equation in an anisotropic body with thermal diffusivity proportional to the polynomial into square brackets of the right term of the equation (11).

3. THE U.A.L.D. BODY

In the case when only the first two terms of equation (9) are considered, as it follows:

$$T'' = T' + b_2 (T')^2 \quad (12a)$$

or

$$2b_2 T' = \sqrt{1 + 4b_2 T''} - 1 \quad (12b)$$

the body shall be called "Uniformly anisotropic with Linear Diffusivity". Equation (11) becomes indeed:

$$\operatorname{div} \left[\underline{K}_0 \cdot \operatorname{grad} T' \right] + q = c_0 (1 + 2b_2 T') \frac{\delta T'}{\delta t} \quad (13)$$

Equation (13) can be considered as the heat conduction equation in an anisotropic body with diffusivity varying linearly with temperature.

The U.A.L.D. body is then characterized by the two relations (5) and (12), which can be written:

$$\underline{K} = A(T) \underline{K}_0 \quad (14a)$$

$$\int_{T_0}^T \frac{c(T)}{c_0} dT = \int_{T_0}^T A(T) \cdot dT + b_2 \left[\int_{T_0}^T A(T) dT \right]^2 \quad (14b)$$

It can be pointed out that at $T = T_0$:

$$\frac{d}{dT} \left(\frac{c(T)}{c_0} - A(T) \right) = 2b_2$$

It is particularly interesting the case where $\frac{c}{c_0} = 1$ which corresponds to a constant specific heat. In this case, by using the second equation (14), it comes out:

$$2b_2 \int_{T_0}^T A(T) dT = \sqrt{1 + 4b_2 (T - T_0)} - 1$$

which gives:

$$A(T) = \frac{1}{\sqrt{1 + 4b_2 (T - T_0)}} \quad (15)$$

The equation (15) is in fairly good agreement with the experimental data of the heat conductivity for the pyrolytic graphite as presented in Ref. 2.

The case where:

$$\frac{c}{c_0} = 1 \quad (16a)$$

$$\underline{K}(T) = \underline{K}_0 \frac{1}{\sqrt{1 + \frac{T - T_0}{T_*}}} \quad (16b)$$

(being $T_* = \frac{1}{4b_2} = \text{constant}$), the material shall be called $\frac{1}{4b_2}$ U.A.L.D. perfect.

An intermediate case between that described by (14) and that defined by (16), can be obtained by assuming valid the second equation of (16) for the heat conductivity, but without the assumption $T_* = \frac{1}{4b_2}$ and calculating $\frac{c(T)}{c_0}$ by using the second equation (14). The result is the following:

$$\frac{c(T)}{c_0} = \frac{1}{\sqrt{1 + \frac{T - T_0}{T_*}}} \left[1 + 4b_2 T_* \left(\sqrt{1 + \frac{T - T_0}{T_*}} - 1 \right) \right] \quad (17)$$

The material is called U.A.L.D. ideal when:

$$\underline{K} = \underline{K}_0 \frac{1}{\sqrt{1 + \frac{T-T_0}{T_*}}} \quad (18a)$$

$$\frac{c(T)}{c_0} = \frac{1}{\sqrt{1 + \frac{T-T_0}{T_*}}} \left[1 + 4b_2 T_* \left(\sqrt{1 + \frac{T-T_0}{T_*}} - 1 \right) \right] \quad (18b)$$

It can be observed from equations (18) that:

$$T = T_0 + (1 + b_2 T_*) T' \quad (18c)$$

This relation can be particularly useful for the problems where radiation is relevant.

As it can be seen from (18), for homogeneous U.A.L.D. ideal body, when the quantities \underline{K}_0 and c_0 (relative to $T = T_0$) are known, the corresponding variations of \underline{K} with T depend only from the constant T_* , while the similar variation of c with T are functions also of the constant b_2 . For the case $4b_2 T_* = 1$, the (18) become equivalent to (16) and the material is U.A.L.D. perfect.

For the bodies of practical interest \underline{K} decreases in general with the temperature; this means that the constant T_* is actually positive. For the pyrolytic graphite T_* is of the order of 400°F. Referring to (18) it can be seen the way in which $\frac{c}{c_0}$ varies with temperature depends on the non-dimensional constant $4b_2 T_*$.

It gets, indeed:

$$\frac{d \frac{c}{c_0}}{d \left(\frac{T-T_0}{T_*} \right)} = \frac{1}{2} \left[1 + \frac{T-T_0}{T_*} \right]^{-\frac{3}{2}} \cdot (4b_2 T_* - 1) \quad (19)$$

The (19) shows that $\frac{c}{c_0}$ increases with $\frac{T-T_0}{T_*}$ if $4b_2 T_* > 1$, viceversa decreases if $4b_2 T_* < 1$.

4. A TRANSFORM OF THE BOUNDARY CONDITIONS

A transform for the boundary conditions proposed and applied in previous works (Ref. 5 and 6) can be applied also to the anisotropic body. The element of body volume $d\sigma / d\nu$ at the surface σ with the elementary thickness $/d\nu/$ are considered. Each of these elements of volumes acquires, consistently with eq. (4), a heat quantity $Qd\sigma$ for unit of time.

It can be assumed without jeopardizing the correctness of the solution that such quantity of heat instead to entering in the body through σ be locally generated into the element of volume $d\sigma / d\nu$ (Fig. 1).

This is identical to suppose that not heat flux takes place through σ and heat sources q are present in the elements of volume $d\sigma / d\nu$ located in the thickness $/d\nu/$ at the boundary of the body.

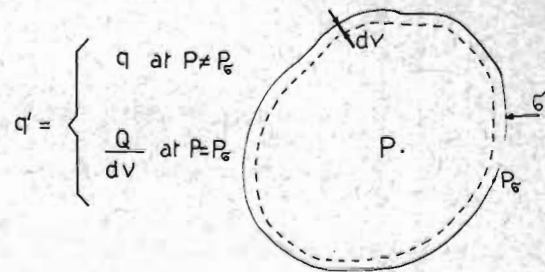


Fig.1 Transform of the boundary conditions

These heat sources q are defined in such way that:

$$d\sigma / d\nu / q = Qd\sigma \text{ or } q / d\nu / = Q$$

As it will be seen the presence of heat sources in the elementary thickness $/d\nu/$ which have an infinite value with finite integral - does not add any analytical difficulty to the integration procedure. Be this transform, the equations (1), (2), (3) and (4) become:

$$t = t_0 \quad T = \tilde{T} \quad (20)$$

$$t > t_0$$

$$- \underline{f} = \underline{K} \cdot \text{grad } T \quad (21)$$

$$- \text{div } \underline{f} + q' = c \frac{\delta T}{\delta t} \quad (22)$$

$$- \underline{f} \cdot \underline{\nu} = 0 \quad (23)$$

In equation (22) it has been written q' instead of q in such a way to include also the heat sources $\frac{Q}{/d\nu/}$ in the thickness $/d\nu/$ at boundary σ .

5. THE HEAT CONDUCTION IN U.A.L.D. BODY

The heat conduction problem in a U.A.L.D. Body can be described by the following relations, where P is a generical point in the body and P_σ a generical point on the surface σ of the body:

A. Assumptions

$$\underline{K} = A \underline{K}_0 \quad c = \frac{c}{c_0} c_0$$

$$\text{Let } A = A(T) \text{ and } \frac{c}{c_0} = \frac{c}{c_0}(T)$$

$$T \int_{T_0}^T A dT \quad T'' = \int_{T_0}^T \frac{c}{c_0} dT = T' + b_2 T'^2 \quad (25)$$

$$q'' = \begin{cases} q \text{ at } P \\ \frac{Q}{/d\nu/} \text{ at } P_\sigma \end{cases}$$

B. Equations

at $t = t_0$ $T' = \tilde{T}'$

at $t > t_0$ $-\underline{f} = \underline{K}_0 \cdot \text{grad } T'$

$-\text{div } \underline{f} + q' = c_0 (1+2b_2 T') \frac{\delta T'}{\delta t}$ (26)

$\underline{f} \cdot \underline{\nu} = 0$ (with $\underline{\nu}$ normal to σ at P_σ)

Now it is considering the function U_n of P as defined, unless a constant multiplier, by the relations:

$\text{div}(\underline{K}_0 \cdot \text{grad } U_n) + p_n^2 c_0 U_n = 0$ at P

$(\underline{K}_0 \cdot \text{grad } U_n) \cdot \underline{\nu} = 0$ at P_σ

with $n = 0, 1, 2, \dots$

and p_n^2 a positive constant to be determined (problem's eigenvalues).

The U_n are making up a closed and complete set of orthogonal functions. If the constant multiplier is known, the basic property of the U_n is the following:

$$\int_V c_0 U_n U_m dV = \begin{cases} 0 & \text{if } n \neq m \\ 1 & \text{if } n = m \end{cases} \quad (28)$$

In the case of bodies of simple shapes (like cylinder, sphere, parallelepiped, etc.) which are homogeneous with oleotropic anisotropy the functions U_n are known (Ref. 1). In a more general case many and very good numerical methods are available to calculate the U_n with the desired accuracy. Further-ly the constant p_n and the functions U_n defined by (28) are taken as known.

Coming back now to system (25) it is assumed that T' and q' might be written in the following form:

$T' = \sum_{n=0}^{\infty} v_n(t) U_n(P)$ (29)

$q' = c_0 \sum_{n=0}^{\infty} g_n(t) U_n(P)$ (30)

From inspection of equations (27) and (28) it can be seen that for $n=0$ $p_0=0$ and $U_0 = (\int_V c_0 dV)^{-1/2}$ furthermore for $n > 0$, $\int_V c_0 U_n dV = 0$.

Substituting now (29) and (30) in the system (25), by using eq. (28), it obtains:

$\frac{dv_n}{dt} + p_n^2 v_n = g_n - 2 b_2 \sum_{i,j} A_{i,j} \frac{d}{dt} (v_i v_j)$ (31)

let $t = t_0$ $v_n = (v_n)_0 = 0$

Letting $\sum_{i,j} = \int c_0 U_i U_j U_n dV$ (32)

$\sum_{i,j} =$ to be extended to all the combinations without repetition of i and j .

From equations (31) it is possible to get the

integral equations: (33)

$$v_n = e^{-p_n^2(t-t_0)} \left[\int_{t_0}^t \left[g_n - 2b_2 \sum_{i,j} A_{i,j} \frac{d}{dt'} (v_i v_j) \right] e^{p_n^2(t'-t_0)} dt' \right]$$

The equations (33) can be integrated step by step starting from the initial conditions which are known at $t=t_0$. In equation (33) it can be also taken into account the heat radiation from the body, by employ- ing equation (18c).

6. SIMILARITY LAWS FOR THE THIN SPHERE

The spherical shell is assumed to be homogeneous and transversally isotropic, id est oleotropic with two principal conductivities only.

Be r, β, α the polar spherical coordinates and $K_0 \cdot A(T)$ and $K_1 \cdot A(T)$ the conductivities in the r and in the β or α directions respectively. It r_0 and r_1 are the outer and inner radius, respectively, it is assumed that $s = r_0 - r_1 \ll r_0$. With such an as- sumption the heat conduction equation simplifies as follows:

$$\frac{\delta^2 V}{\delta \xi^2} + \frac{\lambda}{\cos \beta} \frac{\delta}{\delta \beta} \left[\cos \beta \frac{\delta V}{\delta \beta} \right] + \frac{\lambda}{\cos^2 \beta} \frac{\delta^2 V}{\delta \alpha^2} + p' = (1 + \mu \frac{V}{2}) \frac{\delta V}{\delta \tau}$$
 (34)

where the initial conditions are

at $t = t_0$ $V = 0$ (35)

and the boundary conditions are:

$\xi = 0$ $\frac{\delta V}{\delta \xi} = 0$ (36)

$\xi = 1$ $\frac{\delta V}{\delta \xi} = 0$

In eq. (34) we have:

$V = 2 \left[\sqrt{1 + \frac{T-T_0}{T_*}} - 1 \right]$ (37)

or $\frac{T-T_0}{T_*} = V \cdot (1 + \frac{V}{4})$ (38)

(39) $\xi = \frac{r_0-r}{r_0-r_1}$ $p' = \frac{q' s^2}{K_0 T_*}$ (40)

(41) $\lambda = \frac{K_1}{K_0} \frac{s^2}{r_0^2} = \text{const.}$ $\mu = 4b_2 T_*$ (42)

$\tau = t \frac{K_0}{c_0 s^2}$ (43)

Integration of (34), (35), (36) for example by using the method outlined previously, gives all so- lutions coming out from the similarity laws (37)- (43).

A crude approximation can be used to linearize the eq. (34).

Let us assume that $\Phi = \frac{Qs}{K_0 T_*}$ is the non dimen- sional heat flux

at $\xi = 0$, and be $q = 0$. Be:

$$\bar{P} = \frac{1}{\sigma} \int_0^{Qs} \frac{Qs}{K_0 T_*} d\sigma = \bar{P}(\tau) \quad (44)$$

the average heat flux at $\xi = 0$.

A solution of eq. (34) can be found which is dependent on τ only:

$$\bar{P} = (1 + \mu \frac{\tilde{V}}{2}) \frac{d\tilde{V}}{d\tau} \quad (45)$$

Such a solution is correct only if \bar{P} is distributed uniformly with body. It is obtained:

$$1 + \mu \frac{\tilde{V}}{2} = \sqrt{1 + \mu \int_0^{\tau} \bar{P} d\tau} \quad (46)$$

If in the equation (34) to the expression $(1 + \mu V/2)$ the approximation (46) is substituted, it is obtained the linear equation:

$$\frac{\delta^2 V}{\delta \xi^2} + \frac{\lambda}{\cos \beta} \frac{\delta}{\delta \beta} \left[\cos \beta \frac{\delta V}{\delta \beta} \right] + \frac{\lambda}{\cos^2 \beta} \frac{\delta^2 V}{\delta \alpha^2} + p' = \frac{\delta V}{\delta \vartheta} \quad (47)$$

where

$$\frac{d\tau}{d\vartheta} = \sqrt{1 + \mu \int_0^{\tau} \bar{P} d\tau} \quad (48)$$

Eq. (47) with (35) and (36) can be solved very easily, because in this case the eigenfunction U_n and the eigen-values p_n^2 are perfectly known (Ref. 5).

Once eq. (47) with (35) and (36) are solved for a given value of λ and for given \bar{P} , we know $V = V(\xi, \beta, \alpha, \vartheta)$. In order to come back to the dimensional problem, the following formula:

$$\tau = \vartheta + \frac{\mu}{2} \int_0^{\vartheta} d\vartheta \int_0^{\vartheta} \bar{P}(\vartheta) d\vartheta \quad (49)$$

and (38)-(43) can be used.

7. THE U.A.L.D. PARALLELEPIPED

By using also on the case eqs. (37), (40)-(43) it is obtained:

$$\frac{\delta^2 V}{\delta \xi^2} + \lambda \left(\frac{\delta^2 V}{\delta \eta^2} + \frac{\delta^2 V}{\delta \zeta^2} \right) + p' = (1 + \mu \frac{V}{2}) \frac{\delta V}{\delta \tau} \quad (50)$$

$$\xi = 0 \quad \frac{\delta V}{\delta \xi} = 0$$

$$\xi = 1 \quad \frac{\delta V}{\delta \xi} = 0 \quad \text{where} \quad \xi = \frac{x}{s}$$

$$\eta = \eta_0 \quad \frac{\delta V}{\delta \eta} = 0$$

$$\eta = \eta_1 \quad \frac{\delta V}{\delta \eta} = 0 \quad \text{where} \quad \eta = \frac{y}{r_0}$$

$$\zeta = \zeta_0 \quad \frac{\delta V}{\delta \zeta} = 0$$

$$\zeta = \zeta_1 \quad \frac{\delta V}{\delta \zeta} = 0 \quad \text{where} \quad \zeta = \frac{z}{r_0}$$

being $s = x_0 - x_1$ and r_0 a length of reference in the yz plane.

The general method of solution obtained in the report can be applied very easily, because p_n^2 and U_n are well known.

A crude approximation, similar to the one used for the thin sphere, can be worked out.

8. THE U.A.L.D. THIN CIRCULAR CYLINDER

Be r, ω, z the cylindrical coordinates; it is assumed $s = r_0 - r_1 \ll r_0$.

It is obtained:

$$\frac{\delta^2 V}{\delta \xi^2} + \lambda \left(\frac{\delta^2 V}{\delta \omega^2} + \frac{\delta^2 V}{\delta \zeta^2} \right) + p' = (1 + \mu \frac{V}{2}) \frac{\delta V}{\delta \tau} \quad (51)$$

$$\xi = 0 \quad \frac{\delta V}{\delta \xi} = 0 \quad \text{where} \quad \xi = \frac{x}{s}$$

$$\xi = 1 \quad \frac{\delta V}{\delta \xi} = 0$$

$$\zeta = \zeta_0 \quad \frac{\delta V}{\delta \zeta} = 0$$

$$\zeta = \zeta_1 \quad \frac{\delta V}{\delta \zeta} = 0 \quad \text{where} \quad \zeta = \frac{z}{r_0}$$

Similarity rules are the same as for thin sphere, exposed by eq. (37)-(43).

The general method of integration can be used for (51).

Also in this case p_n^2 and U_n are well known (Ref. 5).

Also in this case a crude approximate solution can be immediately found by using the same procedure already seen for the sphere.

9. HEAT RADIATION IN THE U.A.L.D. BODY

In eq. (4), Q includes the radiation losses Q_r :

$$Q_r = \Omega \sigma T^4 \quad \text{or}$$

$$Q_r = \Omega \sigma \left[T_0 + T' \left(1 + \frac{r'}{4 T_*} \right) \right]^4 \quad (52)$$

By introducing (52) into eq. (31) it is obtained:

$$g_n = \bar{g}_n + \Omega \sigma \int_{\sigma} \left\{ T_0 + \left(\sum_r V_r U_r \right) \left[1 + \frac{1}{4 T_*} \sum_s V_s U_s \right] \right\}^4 U_n d\sigma \quad (53)$$

where $\bar{g}_n = \int_{\sigma} Q_a U_n d\sigma$ is assumed to be a known function of time and $Q = Q_a - Q_r$.

Expression of eq. (53) supplies:

$$g_n = \bar{g}_n + \Omega \sigma S(V_r) \quad (54)$$

where S is a polynomial expression of the eight order in the unknowns V_r . The nine coefficients of S are known surface integrals of products of the eigenfunctions U_n . Introducing (54) into eq. (33), the effect of surface radiation is fully taken into account.

10. GENERALIZATION OF THE METHOD

The most of this report deals with a U.A.L.D. body.

It is obvious that by starting from equation (11) it is possible to generalize the procedure to a body with uniform anisotropic and polynomial diffusivity.

11. NUMERICAL EXAMPLE

As an application of previous analysis the case of a re-entering anisotropic hollow sphere is considered.

Initial altitude is supposed of 80 km, and a straight-line trajectory is considered, with a flight path angle of 12° .

Outer radius of the sphere is 0.5 m, thickness is 0.0057 m, and the weight is 50 kg.

The thermal properties of the anisotropic material considered (pyrolytic graphite) (Ref. 2) are such that it is possible to assume $K_1/K_0 = 160$. Fig. 2 shows the differences between the real values of $K(T)/K_0$ and those supposed in the calculation.

Calculation of heat flow rates was performed by the method of Ref. 4 and results are plotted in Fig. 3 at the stagnation point.

The variation of heat transfer coefficient, referred to stagnation point value, is given in Fig. 4. The said distribution is quite accurately expressed as

$$0.3975 + 0.6942 P_2 - 0.0917 P_4$$

where P_2 and P_4 are Legendre's polynomials of order 2 and 4 respectively.

As said, the calculation is performed by using the formulas of the previous part.

Fig. 5 shows time-history of temperature at the stagnation point and at corresponding point of the inner surface.

It is possible to see that the temperature of the surface increases until the stabilized temperature without reaching any peak of temperature as on the outer surface.

Fig. 6 shows the variation of temperature along radius corresponding to the stagnation point at maximum time.

Fig. 7 shows the variation of temperature at outer surface at maximum temperature time.

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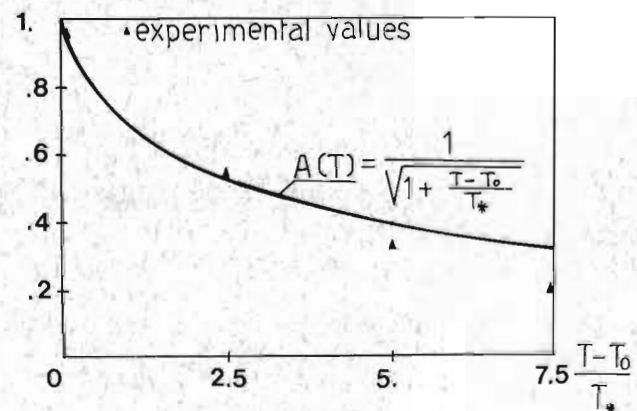


Fig.2 Comparison between real and experimental values of $K(T)/K_0$

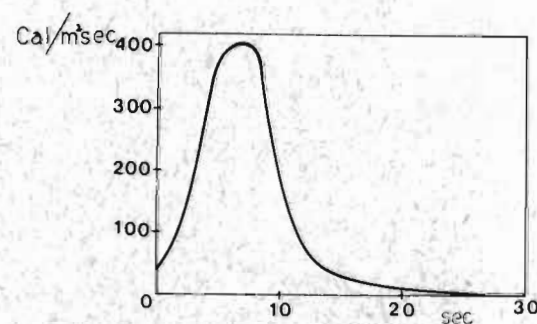


Fig.3 Timewise variation of surface heat flow rate at stagnation point

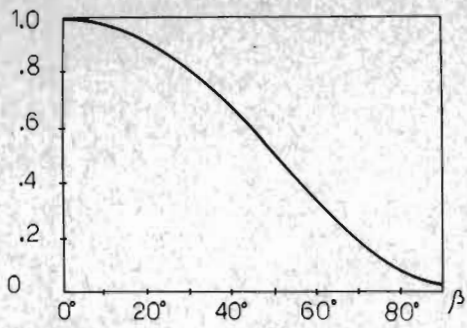


Fig.4 β -wise variation of heat transfer coefficient (referred to stagnation point)

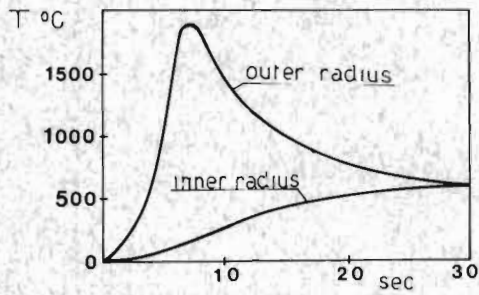


Fig.5 Time history for different radii at stagnation point in the U.A.L.D. spherical shell

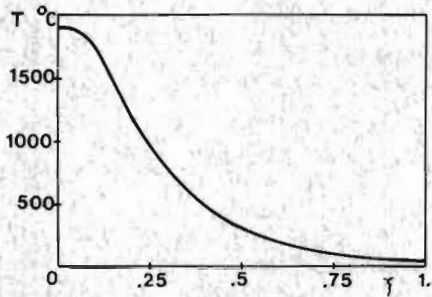


Fig.6 r -wise variation of temperature along radius corresponding to stagnation point (at maximum temperature time)

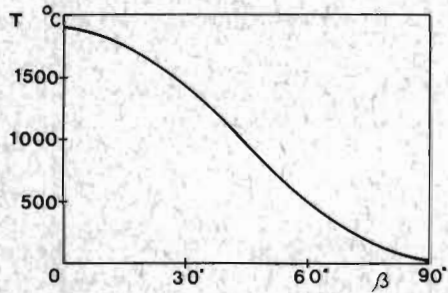


Fig.7 β -wise variation of temperature at outer surface at maximum temperature time